Laminar mixed convection heat transfer to viscoelastic fluids in a 5:1 rectangular channel

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The fully established friction factors and heat transfer coefficients of the laminar flow of viscoelastic fluids in a 5:1 rectangular duct were measured experimentally. The bottom wall, one of the longer sides of the duct was heated by passing direct electric current through it, yielding constant peripheral wall temperature and constant heat input per unit length along the bottom wall. The other surfaces were adiabatic. The test section is 150 hydraulic diameters long. Aqueous solutions of two different polymers (polyacrylamide and hydroxyethylcellulose) were used as viscoelastic fluids.

The measured laminar friction factors agree well with the Newtonian correlation when plotted as a function of Kozicki generalized Reynolds number (i.e., $f=16/\mathrm{Re}^*$). The established laminar Nusselt values for aqueous solutions of hydroxyethylcellulose, a weakly elastic fluid, are not significantly different from the values for water when compared at about the same values of Prandtl number and Rayleigh number. The Nusselt numbers for polyacrylamide solutions are about 25% higher than those for water at comparable values of Prandtl number and Rayleigh number.

Keywords: mixed convection; viscoelastic fluids; rectangular channels; laminar heat transfer

Introduction

The hydrodynamics and heat transfer behavior of non-Newtonian fluids in laminar flow is of special interest because of the wide applications of such fluids in food, chemical and oil industries. Consequently, some analytical and experimental work has been carried out on this topic. The majority of such analytical investigations deal with simpler geometries of flow passages such as circle and plane parallel plates. NonNewtonian fluids are characterized by the nonlinear dependence of shear stress on shear rate. The governing equations for hydrodynamics and heat transfer of nonNewtonian fluids may be found in Refs. 1 and 2. For simple viscometric flow of a power-law fluid the following relationship between shear stress and shear rate is used

$$\tau = K\dot{\gamma}^n = \eta\dot{\gamma} \tag{1}$$

where γ is the shear rate for viscometric flow. Kozicki *et al.*[3] introduced a generalized Reynolds number, Re* such that the fully established friction factor for nonNewtonian laminar flow through any constant cross section channel is given by the well known circular pipe correlation

$$f = 16/\mathrm{Re}^* \tag{2}$$

$$\mathrm{Re}^* = \frac{\rho v^{2-n} D_h^n}{K[(a+bn)/n]^n 8^{n-1}} \tag{3}$$

The rheological constants in n and K may be obtained using Equations 5 and 6. Kozicki and Tiu[4] subsequently improved their analysis for the laminar friction factor in ducts of arbitrary cross section and presented the geometric parameters for rectangular ducts in numerical and graphical forms. The values a and b for a 5:1 rectangular duct are about 0.3 and 0.9, respectively. The published experimental friction factor data for nonNewtonian laminar flow through rectangular channels[5,6] are in good agreement with Equation 2. Some analytical and experimental studies suggest that the Newtonian laminar friction factor is higher than the isothermal value when free convection is present in the flow channel.[7–10] These investigations also reveal that the departure of the friction factor from its isothermal value occurs at a higher value of the Graetz number as the Rayleigh number increases. No information about the buoyancy effect on laminar friction factor for nonNewtonian fluids appears to be published.

Very limited analytical[11–14] and experimental[14–16] studies on laminar heat transfer to nonNewtonian fluids in rectangular ducts are published. For power-law fluid flow through rectangular passage of given aspect ratio the laminar Nusselt value is a function of the Graetz number and n for fully developed velocity conditions at start of heating. For simultaneous developing flow the laminar Nusselt value depends also on the Prandtl number in addition to Graetz and power-law exponent n.

Mena *et al.*[15] experimentally observed a considerable increase in heat transfer to viscoelastic fluids in rectangular duct compared with the circular pipe at a fixed flow rate. Similar results were reported by Oliver[17] and Oliver and Karim[18] in flattened tubes. Kostic[19] reported his experimental results of laminar heat transfer to viscoelastic fluids in a 2:1 rectangular duct. The enhancement in heat transfer is attributed to the possible secondary flow caused by the unequal normal forces on the fluid boundary.

The limited experimental data for laminar heat transfer to nonNewtonian fluids in rectangular ducts were generally forced convection dominated and few experimental studies of free convection effects are available. There are no data published for the case when only one wall is heated.

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Experimental program

The flow passage has a rectangular cross section (2.25 cm × 11.25 cm) of aspect ratio 0.2. The length of the duct is 150 hydraulic diameters. Details of construction of the test section may be found in Ref. 20. A sharp entrance is provided at the beginning of the test section such that the flow develops from the entrance. The top and side walls were made of acrylic plastic whereas the bottom wall is made of stainless steel (type 304). Heating of the test section lower wall starts from the entrance. Twenty pressure taps are located along the main flow direction and four pressure taps are located peripherally at 125 hydraulic diameters from the entrance. Some 83 thermocouples are cemented on the bottom of the lower wall and 8 thermocouples are embedded in the side walls and 17 in the top wall. The upstream calming section and downstream mixing chamber were made of acrylic plastic cylinders. The test section is insulated with fiberglass wool.

The schematic of flow loop is shown in Figure 1. The flow loop consists of a 400-gal plastic reservoir, two auxiliary tanks of 55 gal each, a positive displacement pump, a test section, a heat exchanger and a weighing tank. The test loop could be operated in a recirculation mode and in a once-through mode. For the heat transfer runs in the recirculation mode the test fluid exiting the test section can either pass through or bypass the heat exchanger before returning to the auxiliary tank. In the once-through mode the fluid is discharged into the drainage system after passing through the weighing tank. Provision is also made to run the loop with gravity feeding in order to avoid the effect of pump oscillations especially for small flow rates. Only PVC piping and valves were used in the entire flow loop to minimize any possible chemical reaction between the test fluid and various components of flow loop.

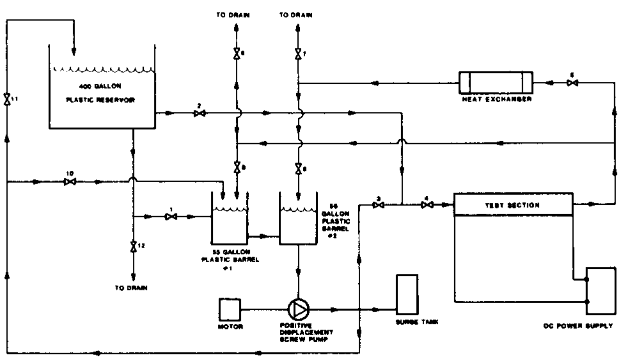

Figure 1 Schematic of flow loop

The rheology of aqueous polymer solutions is sensitive to the solute and solvent chemistry, mechanical degradation, thermal and biological degradation. In light of this all the test fluids studied were prepared in the 400-gal plastic reservoir by dissolving the polymer powder of calculated weight in tap water at room temperature. The viscoelastic fluids studied were aqueous solutions of polyacrylamide (Separan AP-273 produced by Dow Chemicals, Inc.) and Hydroxyethylceullose (Natrosol HHR 250 produced by Hercules, Inc.). Each of these polymers came from a single batch. To minimize mechanical degradation the test fluids were prepared using a hand stirrer for mixing rather than using a mechanical mixer. For all the laminar runs only gravity feeding was employed. Furthermore, it was necessary to operate the system in a recirculation mode as the aqueous solution in the reservoir was insufficient to achieve a steady state condition (e.g., thermal equilibrium). It would take, normally, about 25 to 30 minutes before such a steady state was reached. The maximum available hydraulic head for gravity feeding in the present system is only about 90 cm of water column. The test fluid exiting the test section was returned to the reservoir by either manually bucketing or pumping back to the reservoir depending on the flow rate. In such cases the hydraulic head in the reservoir was maintained at a constant level to within ± 3 cm (which corresponds to about 2% change in the flow rate). It was found that the pressure drop across the heat exchanger is many times greater than the pressure drop across the test section. Therefore, the heat exchanger was

Notation

a	Constant defined in Equation 3
A	Area of cross section of channel, m^2
b	Constant defined in Equation 3
c_p	Specific heat of test fluid, kJ/kg K
dp	Pressure drop, Pa
dx	Distance between the pressure taps, m
D_h	Hydraulic diameter of channel, m
h	Local heat transfer coefficient, $q''/(T_w - T_b)$, W/K m^2
k_f	Thermal conductivity of test fluid, W/m K
K	Consistency index for power-law fluids
L_{th}	Thermal entrance length, m
n	Power law exponent
q''	Uniform heat flux through lower heated wall, W/m^2
T_b	Fluid bulk temperature at any axial position, °C
v	Fluid mean axial velocity, m/s
x	Axial coordinate along channel length, m

Greek symbols

α^*	Aspect ratio of rectangular channel, ratio of shorter side to the longer side of flow cross section
β	Volume coefficient of expansion, 1/K
η	Apparent viscosity of polymer solution $= \tau_w/\dot{\gamma}$, kg/(m·s)
$\dot{\gamma}$	Shear rate, 1/s
ν	Kinematic viscosity, η/ρ, m^2/s
ρ	Density of test fluid, kg/m^3
$\tau_{11} - \tau_{22}$	First normal stress difference, Pa
τ_w	Wall shear stress, $(-D_h/4)(dp/dx)$, Pa

Dimensionless quantities

f	Fanning friction factor $= \tau_w/(\rho v^2/2)$
f_m	Peripherally and axially averaged Fanning friction factor
Gr_q	Modified Grashof number $= g\beta q'' D_h^4/k_f \nu^2$
Gz	Graetz number, $Pe/(x/D_h)$
L_{th}^*	Dimensionless thermal entrance length, L_{th}/D_h Pe
Nu	Peripherally averaged local Nusselt value, hD_h/k_f
Pe	Peclet number $= Re_a Pr_a$
Pr_a	Prandtl number based on apparent viscosity, $\eta c_p/k_f$
Ra_q	Modified Rayleigh number $= Gr_q Pr_a$
Re_a	Reynolds number based on apparent viscosity, $\rho v D_h/\eta$

bypassed when gravity feeding was used so as to have better control over flow rate.

After turning on the flow through the test section all the air bubbles were thoroughly vented before turning on the power supply for heating the channel. The heat input was gradually increased until the temperature difference between the heated wall and the fluid bulk is about 2.5 to 3°C (at $x/D_h \sim 100$). The inlet bulk temperature was kept very close to the room temperature by adjusting the cooling water flow through the heat exchanger. All the thermocouple readings, voltage drop across the heated wall, current through the test section, pressure drop in the fully developed region ($x/D_h > 100$), fluid bulk temperature at inlet and exit of the test section and mass flow rate were taken for each run after steady state (thermal equilibrium) is reached. Measurement of the fluid temperatures at the inlet and exit of the test section combined with the measured mass flow rate provides an independent check on the measured heat input.

The inside temperature of the lower heated wall is calculated using the one-dimensional conduction approach. Axial and peripheral heat conduction in the steel wall is neglected. For aqueous polymer solutions studies all of the thermophysical properties except viscosity are taken to be the same as for water.[21] The steady shear viscosity of the polymer solutions was obtained from the Weissenberg rheogoniometer by measuring the wall shear stress and shear rate.

It was assumed that the relationship between the wall shear stress and shear rate may be approximated by the following equation:

$$\ln \tau_w = a_0 + a_1[\ln \dot{\gamma}] + a_2[\ln \dot{\gamma}]^2 \tag{4}$$

The power-law exponent and the consistency index are given by the following expressions:

$$n = d[\ln \tau_w]/d[\ln \dot{\gamma}] = a_1 + 2a_2 \ln \dot{\gamma} \tag{5}$$

$$K = \exp[a_0 - a_2(\ln \dot{\gamma})^2] \tag{6}$$

Characteristic curves of the apparent viscosity are given as a function of the shear rate in Figure 2. The first normal stress difference for various concentrations of aqueous solutions of polyacrylamide (Separan) are shown in Figure 3. The rheology of each test fluid was measured before and after each run and the results were found to be within 2 to 3%.

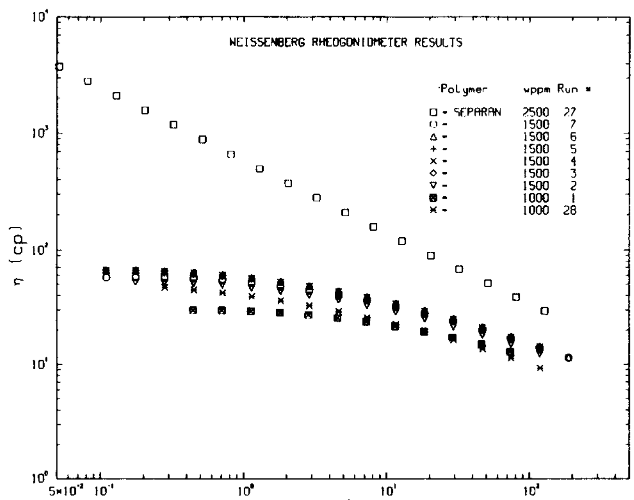

Figure 2 Characteristic curves for viscoelastic fluids

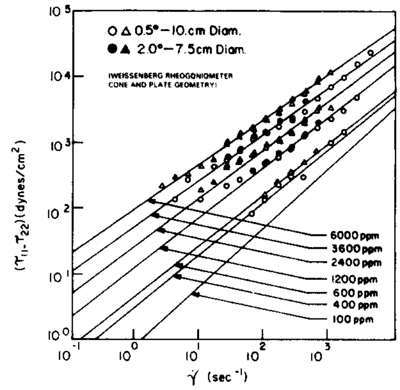

Figure 3 First normal stress difference for aqueous solutions of polyacrylamide (Separan)

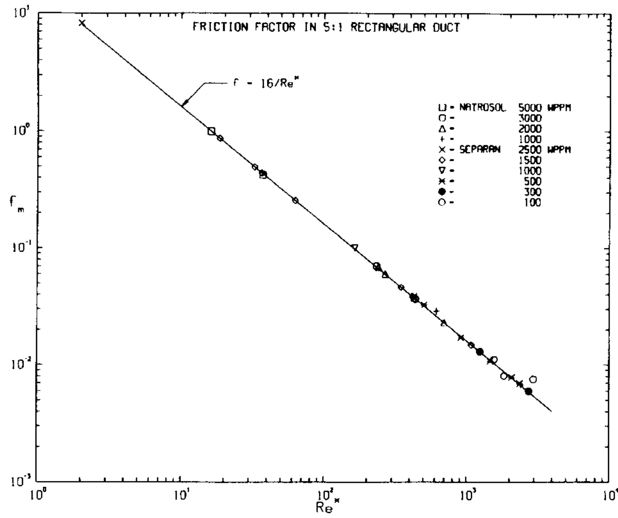

Figure 4 Laminar friction factor vs. Kozicki generalized Reynolds number

Results

The pressure drop measurements were carried out along with the heat transfer measurements. The measured fully established laminar Fanning friction factors for the aqueous polymer solutions studied are shown as a function of Kozicki generalized Reynolds number in Figure 4. The results show good agreement with the Newtonian correlation for isothermal laminar flow. The present laminar friction factors do not show any significant deviation from the isothermal values. The estimated uncertainty values for friction factors and Reynolds numbers in this study are within ±5% and ±2%, respectively.

In the case of a rectangular geometry, where there are four bounding walls many different thermal boundary conditions are possible. For example all four walls can be maintained at a constant wall temperature (the *T* boundary condition). Alternatively, a constant heat flux can be imposed axially on all four walls, with constant temperature peripherally (the H1 boundary condition), or a constant heat flux can be imposed on all four walls, both axially and peripherally (the H2 condition). These same conditions can be imposed on one or more of the four walls, with the other walls insulated. In the present study H1 thermal boundary condition is imposed on the lower wall while the other walls are adiabatic. The boundary condition for the present case is denoted by H1(1), meaning

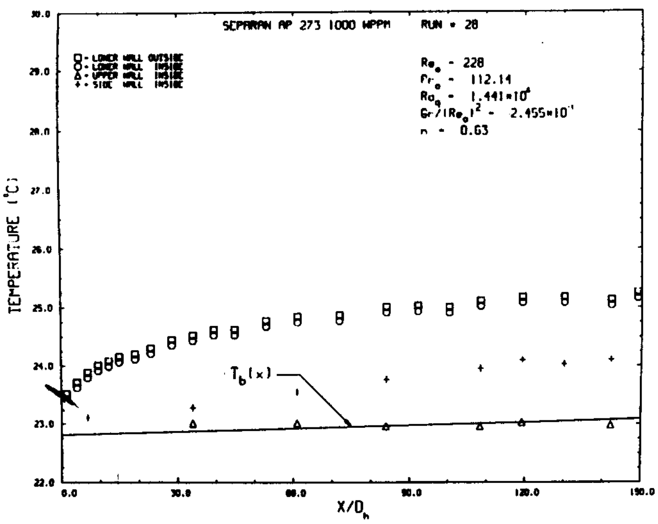

Figure 5 Measured axial temperature distribution for a typical Separan run

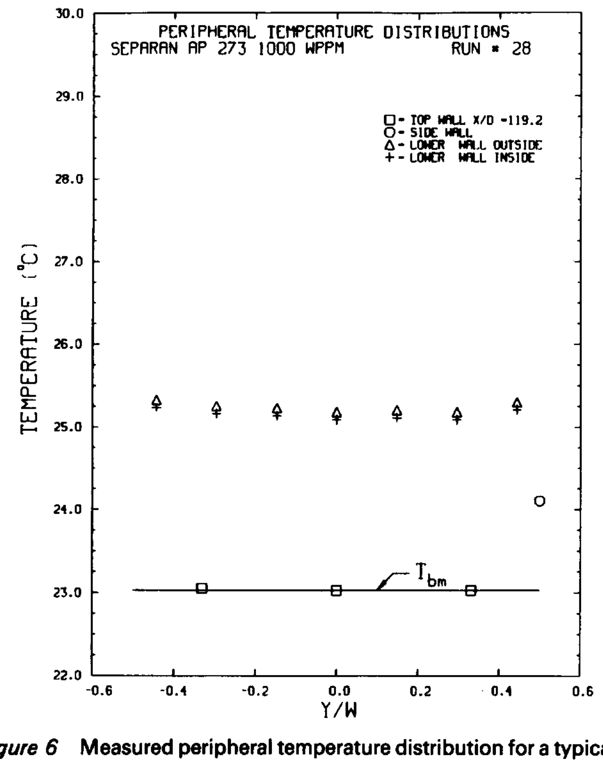

Figure 6 Measured peripheral temperature distribution for a typical Separan run

that only one of the wider walls is heated. The measured wall temperature distributions for a typical laminar run are shown in Figures 5 and 6. The spanwise temperature distribution of the heated wall is relatively flat suggesting that one-dimensional conduction approach to calculate the inside temperature of the heated wall is reasonably accurate. The measured laminar Nusselt values for typical runs of polyacrylamide (Separan) solutions and hydroxyethylcellulose (Natrosol) solutions are shown as a function of Graetz number in Figures 7 and 8. Using the values for dimensionless thermal entrance lengths for rectangular passages[22] a value of L_{th}^{*} of 0.8 is estimated for the 5:1 geometry for the H1(4) condition. In general it is anticipated that the H1(1) condition will require a greater thermal entrance length than the H1(4) condition. It can be concluded that the present experimental Nusselt values for the viscoelastic fluids would be in the thermal entrance region under conditions of pure forced convection. However, the presence of free convection appreciably decreases the thermal entrance length. The experimental temperature profiles and local heat transfer coefficients in the case of Separan and Natrosol give clear evidence that the results are in the fully developed thermal regime.

The range of parameters studied is given in Table 1.

The onset of free convection is clearly seen in Figure 7 and the mixed convection Nusselt values appear to have reached fully established values. These results are compared with the equation validated for water[20] at comparable values of the Rayleigh numbers. The fully established laminar Nusselt quantities for Natrosol (known to be weakly elastic) are not markedly different from those of water. The results for Separan (more elastic fluid) are about 20 to 25% higher than those for water at comparable values of Prandtl number and Rayleigh number over the range of Graetz number between 40 and 300. The estimated uncertainty values for Re_a, Pr_a, Gz, and Ra_q were below 2.5%, whereas the uncertainty values for the Nusselt numbers are within $\pm 5\%$.

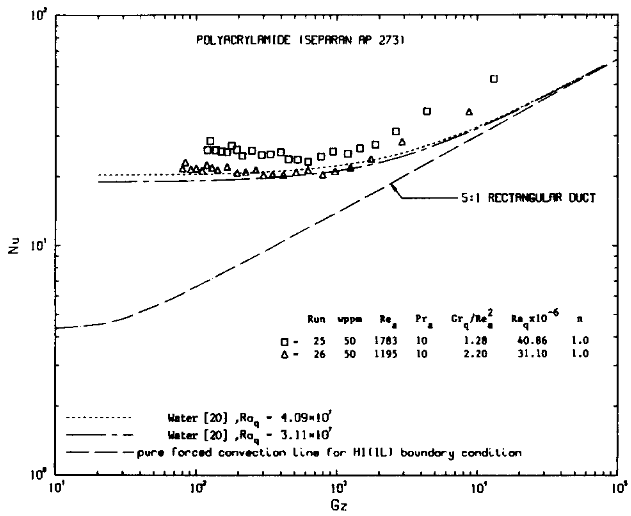

Figure 7 Nusselt value vs. Graetz number for typical Separan runs

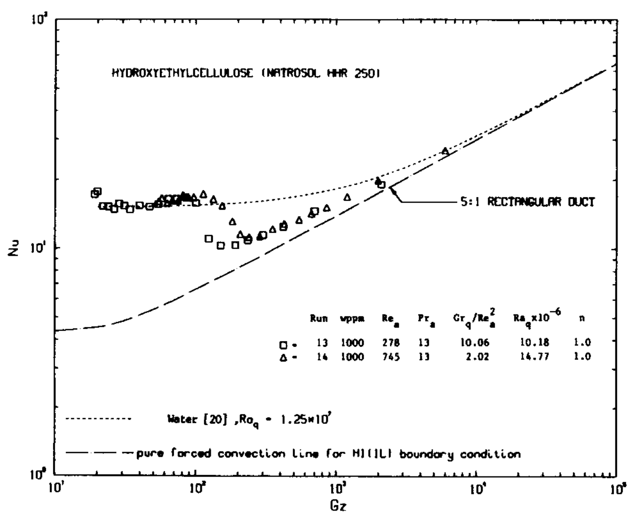

Figure 8 Nusselt value vs. Graetz number for typical Natrosol runs

Table 1

Parameter	Separan	Natrosol
Re_a	3–1783	21–829
Pr_a	10–1275	13–900
Gr_q	$53–4.1\times10^6$	$189–1.1\times10^6$
Ra_q	$7\times10^4–4.1\times10^7$	$1.7\times10^5–1.5\times10^7$
Gr_q/Re_a^2	0.31–5.84	0.03–10.05
q''	300–1640	337–545

Kostic[19] reported experimental laminar Nusselt values for Separan measured in a 2:1 rectangular passage for H1(2) thermal boundary condition. His data showed almost 100% increase in heat transfer when compared with his data for Newtonian fluid (water) at about the same Rayleigh number. The increase in heat transfer to viscoelastic fluid is attributed to the elasticity of the fluid which induces secondary flows in the flow passage (due to the fact that an elastic fluid exerts unequal normal forces on the flow boundaries).

In the present study the laminar Nusselt values measured for Separan are only 20 to 25% higher than those for water. This decrease in heat transfer enhancement is due to the reduced strength of the secondary flow as the aspect ratio decreases.

Conclusions

(1) The fully established laminar friction factors for viscoelastic fluids flowing through rectangular passages can be predicted using the correlation for Newtonian fluids (i.e., $f=16/Re^*$).

(2) The established laminar Nusselt values for aqueous solutions of Natrosol, a weakly elastic fuid, are not markedly different from those of water at a comparable value of Graetz number and Rayleigh number.

(3) Laminar mixed convection Nusselt values for aqueous solutions of Separan, a relatively more elastic fluid, are higher than those for water at comparable values of Rayleigh number and Graetz number. This observed increase in heat transfer is due to the secondary flows induced as a result of the unequal normal forces exhibited by viscoelastic fluids. However, this increase in heat transfer is much smaller than compared with the values reported in the literature for a 2:1 rectangular channel.[19] The secondary flow effects due to elasticity become apparently weaker as the aspect ratio (α^*) decreases.

Acknowledgments

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